Response to Reviewer 2 Comments (round 2)

Open Review	(x)	I would not like to sign my review report
	( )	I would like to sign my review report
English language and style	( )	Extensive editing of English language and style required
	( )	Moderate English changes required
	(x)	English language and style are fine/minor spell check required
	( )	I don't feel qualified to judge about the English language and style

	Yes	Can be improved	Must be improved	Not applicable
Does the introduction provide sufficient background and include all relevant references?	( )	( )	(x)	( )
Are all the cited references relevant to the research?	(x)	( )	( )	( )
Is the research design appropriate?	( )	( )	( )	(x)
Are the methods adequately described?	( )	( )	( )	(x)
Are the results clearly presented?	(x)	( )	( )	( )
Are the conclusions supported by the results?	( )	( )	( )	(x)

In the authors' reply, as well as in the article's text, the claim that the results are new is repeated in a way that makes it straightforward to show it to be questionable. In the Abstract we have the following 2 sentences: ``Some measures of these quantities, such as Shannon entropy and related complexity measures, are defined for objects drawn from a statistical ensemble and cannot be computed for single objects; While some other measures such as Kolmogorov complexity suffers from incomputability. Based on assembly theory, we attempt to fill this gap by introducing the notion of a ladderpath which describes how an object can be decomposed into a hierarchical structure using repetitive elements." as well as ``The ladderpath theory provides a novel characterization of the information that is contained in a single object (or a system)".

What I wish to dispute is both the existence of a ``gap" as argued in the first sentence, as well as the fact that the ladderpath theory provides a ``novel" characterization of the information contained in a single system.

Indeed, the very titles of the references

[1] Abraham Lempel and Jacob Ziv, On the Complexity of Finite Sequences, IEEE Transactions on Information Theory, IT-22, NO. 1, January 1976 pp.~75--81

[2] Jacob Ziv, IEEE Transactions on Information Theory, Vol.~IT-24, No.~4, July 1978, p.~405, Coding Theorems for Individual Sequences
show that an information measure can be defined for an individual sequence, and that it is estimated, as follows from the theorems stated in the paper, by the use of the LZ compression technique as well as related ones. Furthermore, the complexity measure defined in the first of the above references is in fact extremely close to the definition given in the authors' paper, as it also involves the estimation of a minimal procedure to build up the sequence out of its building blocks.

True, in order to prove convergence theorems, one needs infinite sequences, but there is no difficulty of principle in applying these same techniques to large but finite sequences. To be specific, the information measure which is estimated is defined as follows: let $2^{lh_l(u)}$ be the total number of words that appear as you slide a window of size l along the infinite sequence u. It is seen that $\lim_{l\to\infty}h_l(u)=h(u)$ exists, and that its value can be estimated by the LZ compression approach.

Note that the above publications are 35 years old. It is of course understandable that such work, which has not made its way into the textbooks, could be missed. However, ignoring the existence of previous work when rediscovering it, is not the appropriate approach.

Response 1: Thanks for the comment. It is detailed and difficult to disentangle, but two main messages can be summarized: (i) the reviewer said that LZ compression can be used to derive an information measure for individual sequences, no matter finite or infinite; and (ii) the reviewer said that the complexity measure defined in LZ and in ladderpath are extremely close.

For message (i): First of all, we totally agree that LZ compression algorithm can be used to give an information measure for individual sequences, which we never denied. In fact, in the last revision, we have emphasized the importance of LZ algorithm and its relationships with information here and there (e.g., line 753 and 73), as suggested by the reviewer in the last round.

Secondly, in the ladderpath approach, the shortest ladderpath itself is the most important concept (namely the partially ordered hierarchical relationships among ladderons), which can be used to interpret or decode the syntactic information of a sentence, namely the characterization of the structure of the sentence. This is actually an important point of our work, and we discussed it quite often in the manuscript, e.g. in section 1, 2.1, 4.1 and 4.2 (e.g., how to interpret a sequence in an isolated system such as an alien signal, or non-isolated system such as a sentence in a given language). This "syntactic information" (which is our focus) are distinct from what the reviewer referred to (although we agree that LZ can be used to properly estimate the information), which is h(u), a real number that plays a role similar to that of Shannon entropy.

From the shortest ladderpath, indeed some measurement can be abstracted (as a real number, for example) to describe the amount of "information". Then, this measurement would be the counterpart of h(u) in LZ compression, which may or may not converge to h(u). It is actually a very interesting question and deserves further investigations, but it is not the focus of our current paper.

For message (ii): the reviewer raised his or her concern that the complexity measure defined in ref 1 and in ladderpath are "extremely close" because both "involve the estimation of a minimal procedure to build up the sequence out of its building blocks". It is absolutely correct that ladderpath "involves the estimation of a minimal procedure", but that doesn't necessarily mean it's extremely close to LZ compression, because:

1. Ladderpath can deal with not only sequences, but also other objects such as molecules (which we also discussed in the main text, line 717-747), proteins, pictures, etc.
2. There are other approaches and theories that also use the idea of "minimal procedure to build up", e.g., addition chain that applies to integers; but clearly the addition chain also differs from LZ algorithm.
3. Even though we only talk about sequences, the results given by ladderpath and LZ algorithm are also different. We have shown an explicit example in the last response to the reviewer to show the difference, from which it is clear that ladderpath and LZ algorithm are not "extremely close".

Thanks again for the reviewer's comments. Besides the responses, we also slightly revised the wording in the abstract accordingly.

Finally, the answer made to my criticism, that the claim of having ``two axes of complexity'' is in fact illusory, since $\lambda+\omega=S$, where $S$ is the signal length, the authors answer as follows: ``It is true indeed, but in fact, any target can be placed in a particular position in these coordinates. This is because although the three indices are constrained by $\lambda + \omega = S$, two of them are free.'' Fair enough, but when the authors speak of 3 indices, do they really mean that S, the size of the message, is actually an ``index of complexity''? Surely, it seems more natural to normalize both $\lambda$ and $\omega$ by $S$, which leads to the normalized values adding up to one. That 2 values which add up to one do not form ``two axes of complexity'' should be clear.

Response 2: Thanks for these comments.

Indeed, the size-index of the target is closely related to the target's complexity (but note that size-index is not always the number of letters of a sequence; Instead, it's determined by the number of letters and also the basic set, referring to the definition in line 330 in the main text). But the motivation that we choose ladderpath-index and order-index as the axes is that the normal intuition about complexity includes the difficulty to reproduce the system and how order /organized a system is (as we discussed in line 57-64, line 423-431), which correspond to ladderpath-index and order-index, respectively.

The most straightforward evidence and argument why "two axes of complexity" is not illusory is that if we fix the size-index of a target (referring to Figure 2 in the main text), by rearranging the patterns of this target, it can move freely in the coordinates (e.g., imaging rearranging among pattern [i], [iii] and [vi]). That is to say, for a fixed size-index target, the ladderpath-index (also order-index) could be different, which totally depends on the internal structure and information of this target, indicating that ladderpath-index alone cannot be the only measurement of "complexity". These arguments have been added in the last revision (line 432-442).

Moreover, referring to Figure 2 in the main text, if we equate "complexity" to only one index, say, ladderpath-index, then the "complexity" of [i], [ii] and [iv] would be the same, but this clearly conflicts with the intuition. If we equate "complexity" to size-index, then [i], [iii] and [vi] would have the same "complexity", which again conflicts with the intuition.

Thanks for the suggestion or argument of normalization given by the reviewer. But we think they should not be normalized by the size-index S, because the absolute value of the three indices has explicit meanings, for example, larger objects tend to be more "complex" given the same ladderpath-index. If they are normalized, the normalized ladderpath-index of [iv] (6/8=0.75) would be larger than the normalized ladderpath-index of [iii] (11/18=0.61), which does not make much sense and it becomes confusing to interpret the normalized ladderpath-index.

In other words, I remain wholly unconvinced by the authors' arguments and by their answers to my remarks, and stand by my earlier opinion. The paper should be rejected.

Response 3: By this reply and the revision, we hope we have addressed the issues raised by the reviewer. Thanks very much again for your time.